

Indirect interaction of magnetic domain walls

N. Sedlmayr^{*1}, V. K. Dugaev^{2,3}, M. Inglot², J. Berakdar⁴

¹ Department of Physics, University of Kaiserslautern, 67663 Kaiserslautern, Germany

² Department of Physics, Rzeszów University of Technology, Al. Powstańców Warszawy 6, 35-959 Rzeszów, Poland

³ Department of Physics and CFIF, Instituto Superior Técnico, TU Lisbon, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

⁴ Martin-Luther-Universität Halle-Wittenberg, Heinrich-Damerow-Str. 4, 06120 Halle, Germany

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* Corresponding author: e-mail sedlmayr@physik.uni-kl.de, Phone: +49-631-205 2393

We calculate the electron-mediated exchange interaction between two domain walls in magnetic wires. This corresponds to the equilibrium regime and, therefore, the interaction can be additionally controlled by an electric current. The exchange interaction is long ranged and oscillates as a function of the distance between the walls. It also depends oscillatory on the polarization angle of the walls, having the maximum value for collinear polarization.

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1 Introduction Magnetic domain walls (DWs) have attracted a lot of attention as important elements of new magnetoelectronic devices [1,2,3]. In particular, it was demonstrated recently that they can be used in a new type of memory device (the racetrack memory) effectively controlled by an electric current [4,5]. On the other hand, the DW can be viewed as a kind of local imperfection in an ordered magnetic system, like an impurity or defect. The substantial difference is that DWs can move and therefore they can be relatively easily put into motion by an external field or electric current, and also by an interaction between them.

A great amount of theoretical and experimental work was dedicated to studying the resistance of DWs, the current-induced spin torque, the dynamics of DW motion, and other effects related to a single DW strongly coupled to the electron system [6,3,7]. When the density of DWs increases, it is important to include into these considerations the effects of their interaction. It has already been demonstrated [8,9,10,11] that an electric current in a magnetic wire with DWs influences the DW interaction, so that by using a current the DW coupling can be controlled, and the dynamics of strongly coupled DWs can be affected.

In this work we consider in detail the indirect exchange coupling between the DWs in equilibrium. Essentially, the analysis of such interaction is the first necessary step to understand the basic mechanisms of DW interactions.

2 Model Let us consider the following Hamiltonian, describing an electron gas coupled by exchange energy M to a textured magnetization which changes its orientation at each DW:

$$H = -\frac{\Delta}{2m} - M\boldsymbol{\sigma} \cdot \mathbf{n}(\mathbf{r}), \quad (1)$$

where the unit vector $\mathbf{n}(\mathbf{r})$ determines the magnetization orientation, m is the carrier effective mass, and we take units with $\hbar = 1$. We assume the ferromagnetic wire (or ribbon) to be orientated along the x axis and consider two DWs, labeled as 1 and 2. For definiteness, we also assume that at $x \rightarrow -\infty$ and $x \rightarrow +\infty$ the magnetization is along unit vector $\mathbf{n}_0 = (0, 0, 1)$, and within each of the DWs the vector \mathbf{n} is rotated by an angle π around the unit vector \mathbf{t}_i , where the index $i = 1, 2$ refers to DW 1 or 2. We focus here on the case of transverse DWs, the results for a vortex DW should look qualitatively similar. Thus, the dependence of $\mathbf{n}(x)$ within DWs 1 and 2 is described as

$$\mathbf{n}_1(x) = e^{i\varphi_1 \mathbf{t}_1 \cdot \mathbf{L}} \mathbf{n}_0, \quad \mathbf{n}_2(x) = -e^{i\varphi_2 \mathbf{t}_2 \cdot \mathbf{L}} \mathbf{n}_0, \quad (2)$$

where \mathbf{L} is the matrix of moment $L = 1$ and $\varphi_i(x)$ changes from 0 to π when x crosses the i -th DW. Correspondingly, the transformation of the spinor wave function ψ to the local frame with the magnetization along the axis z is $\psi \rightarrow \hat{T}\psi = e^{\frac{i\varphi}{2} \mathbf{t}_i \cdot \boldsymbol{\sigma}} \psi$. The location x_i and the vector \mathbf{t}_i determine the DW state. Note that for the electron motion the DW can be always considered as static even for DWs moving along the wire.

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We use this transformation to the local frame [12,13,14], in which the vector \mathbf{n}_i of each DW is oriented in the same direction along the global axis z . After this transformation the electron gas is in the homogeneous magnetization M but there appears the gauge potential related to the local transformation, $A_i(x) = i(\varphi'_i/2)(\mathbf{t}_i \cdot \boldsymbol{\sigma})$.

We assume the DW width λ to be much larger than the electron wavelength at the Fermi surface, $k_F \lambda \gg 1$, which is the typical condition for metallic ferromagnets. Then the transformed Hamiltonian is (summing over i)

$$H = \frac{k_y^2 + k_z^2}{2m} - \frac{\hat{p}_x^2}{2m} - M\sigma_z - \left[\frac{\beta_i}{2} \mathbf{t}_i \cdot \boldsymbol{\sigma} \hat{p}_x + \text{h.c.} \right], \quad (3)$$

where $\beta_i(x) = \varphi'(x - x_i)/2m$, $\hat{p}_x = -i\partial/\partial x$ and x_i is the point where the i -th DW is located.

The exchange interaction energy can be found using a RKKY approach with the DW-induced perturbation localized in the vicinity of the points $x_1 = 0$ and $x_2 = R$. Using Eq. (3) we find

$$E_{int} = \sigma_{cs} \text{Re Tr} \int \frac{d^2 k}{(2\pi)^2} \frac{d\varepsilon}{2\pi} dx' dx'' \beta_1(x') \beta_2(x'') \times (\mathbf{t}_1 \cdot \boldsymbol{\sigma}) \frac{dG_{k\varepsilon}(x' - x'')}{dx'} (\mathbf{t}_2 \cdot \boldsymbol{\sigma}) \frac{dG_{k\varepsilon}(x'' - x')}{dx''}, \quad (4)$$

where σ_{cs} is the cross-section of the DW and

$$G_{k\varepsilon}(x) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} e^{ik_x x} \text{diag}(G_{\mathbf{k}\varepsilon\uparrow}, G_{\mathbf{k}\varepsilon\downarrow}) \quad (5)$$

is the Green function of electrons in a homogeneous magnetization field: $G_{\mathbf{k}\varepsilon\sigma}^{-1} = \varepsilon - \varepsilon_{k\sigma} - k_x^2/2m + \mu + i\delta \text{sgn}(\varepsilon)$, where μ is the chemical potential. We also denoted $\varepsilon_k = (k_y^2 + k_z^2)/2m$ and $\varepsilon_{k\uparrow,\downarrow} = \varepsilon_k \mp M$. Now we take $k_\sigma = +\sqrt{2m(\varepsilon - \varepsilon_{k\sigma} + \mu) + i\delta \text{sgn}(\varepsilon)}$. Then defining $\xi_{k\sigma} = \varepsilon_{k\sigma} - \mu$ we have the conditions that if $\varepsilon - \xi_{k\sigma} > 0$ and $\varepsilon < 0$ then k_σ lies in the lower half-plane, otherwise it lies in the upper half plane. This allows us to directly calculate $G_{k\varepsilon}(x)$.

Calculating the Green functions and their derivatives in coordinate representation and substituting them into Eqn. (4) we obtain

$$E_{int} = -\sigma_{cs} m^2 \text{Re Tr} \int \frac{d^2 k}{(2\pi)^2} \frac{d\varepsilon}{2\pi} dx' dx'' \times \beta_1(x') \beta_2(x'') (\mathbf{t}_1 \cdot \boldsymbol{\sigma}) \mathbf{P} (\mathbf{t}_2 \cdot \boldsymbol{\sigma}) \mathbf{P}, \quad (6)$$

where we denoted

$$\mathbf{P} = \text{diag}(f_{k\uparrow} e^{ik_\uparrow(x''-x')} + (1 - f_{k\uparrow}) e^{ik_\uparrow(x''-x') \text{sgn}(\varepsilon)}, f_{k\downarrow} e^{-ik_\downarrow(x'-x'')} + (1 - f_{k\downarrow}) e^{-ik_\downarrow(x'-x'') \text{sgn}(\varepsilon)}) \quad (7)$$

and $f_{k\sigma} \equiv f(\varepsilon_{k\sigma})$ is the Fermi-Dirac function at $T \rightarrow 0$. The traces over the matrices can be performed immediately. Assuming that the first DW points into the y -axis, then we can take $\mathbf{t}_1 = \hat{x}$ and $\mathbf{t}_2 = \hat{x} \cos \theta + \hat{y} \sin \theta$, so that

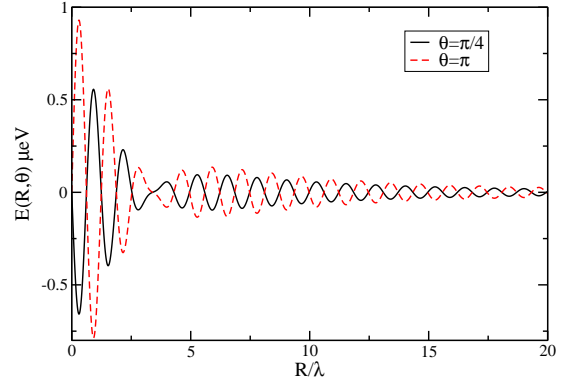


Figure 1 The interaction energy as a function of inter DW distance R , for $\lambda = 10\lambda_F$ and two different angles between the DWs. See text for details.

θ is the angle between the two domain wall polarizations. At $T = 0$ this yields

$$\text{Tr}(\mathbf{t}_1 \cdot \boldsymbol{\sigma}) \mathbf{P} (\mathbf{t}_2 \cdot \boldsymbol{\sigma}) \mathbf{P} = 2 \cos(\theta) \times [f_{k\uparrow} e^{-i(x'-x'')(k_\uparrow+k_\downarrow)} + e^{-i(x'-x'')(k_\uparrow+k_\downarrow \text{sgn} \varepsilon)} + (1 - f_{k\downarrow}) e^{-i(x'-x'')(k_\uparrow+k_\downarrow \text{sgn} \varepsilon)}]. \quad (8)$$

For the β function we use $\beta_1(x) = \text{sech}(x/L)/2mL$, then we can calculate the integrals over dx' and dx'' using

$$\int dx' \beta_i(x') e^{\mp i k x'} = \frac{\pi}{2m} \text{sech} \frac{\pi L k}{2} e^{\pm i k R_i} \quad (9)$$

for $i = 1, 2$ and $R_1 = 0$, $R_2 = R$. Now we can switch the k -integrals to polar coordinates and perform the angular integral, and then substituting $\varepsilon' = k^2/2m$ and rescaling $\varepsilon = \varepsilon - \varepsilon' + \mu$ we obtain

$$E_{int}(\theta, R) = -\frac{m\sigma_{cs} \cos \theta}{32} \text{Re} \left[\int_0^\infty d\varepsilon' \int_M^\infty d\varepsilon e^{i R k_1} \times \text{sech}^2 \frac{\pi L(k_1 + k_2)}{2} \left(e^{i R k_2} + e^{i R k_2 \text{sgn}(\varepsilon + \varepsilon' - \mu)} \right) + 2M \int_M^\infty d\varepsilon e^{i R(k_1 + k_2)} \text{sech}^2 \frac{\pi L(k_1 + k_2)}{2} \right], \quad (10)$$

where $k_{1,2} = \sqrt{2m(\varepsilon \pm M)}$.

The dependence of the exchange interaction on the distance for the two polarization angles θ is shown in Fig. 1. Here we use the following parameters: the Fermi wavelength $\lambda_F = 0.367$ nm, $M = 36$ meV, $\sigma_{cs} = 20 \times 20$ nm², and $\lambda = 10\lambda_F$. The magnitude of the interaction depends strongly and non-monotonically on the DW width, λ , and the magnetization strength, M . They are related by the strengths of the anisotropy and the exchange energy in the material. Halving the width of the DW to $\lambda = 5\lambda_F$, we already see a much larger effect, see Fig. 2.

The obtained results show that the interaction of two DWs is long-ranged and strongly oscillating. The particular behaviour around $R \approx 4\lambda$ is caused by the change of

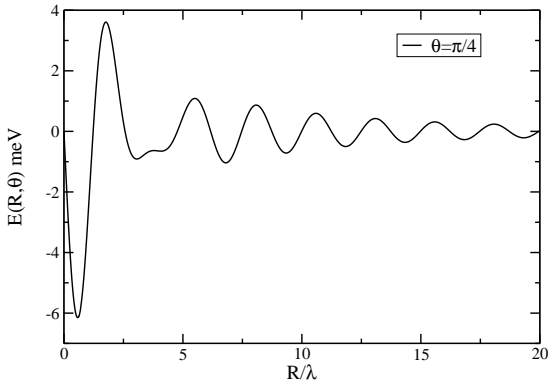


Figure 2 The interaction energy as a function of inter DW distance R , for $\theta = \pi/4$ and $\lambda = 5\lambda_F$. See text for details.

sign of the envelope function of the energy. I.e. the envelope function is also not a monotonic function of R . But the most important is that the DW interaction depends on both the distance between the walls and the DW polarization determined by the angle θ . It means that if we put DWs located at a certain distance R , which corresponds to the energy-favorable collinear mutual polarization of DWs, then there is another location very close in R with the anti-collinear orientation of DWs, with almost the same energy. Considering the energy of the system as a function of distance R , we find the correspondence to a classical “particle” in the oscillating potential profile, so that the neighboring positions of this particle in the minima of the potential describe the up and down states of one of the DWs with respect to the other one. As the amplitude of interaction increases with decreasing R at small distances (see Figs. 1 and 2), the DWs are effectively attracted to each other.

One can assume that one of the DWs is not moving (e.g., due to pinning). Let us assume that the classical particle representing the other DW is located in one of the potential minima. If the distance between the minima is small, the particle can tunnel through the barrier, so that the other DW can be presented as a delocalized quantum particle. The Hamiltonian which describes it, has the form $H = t \sum_i (c_{i\sigma}^\dagger \sigma_{\sigma\sigma'}^x c_{i+1,\sigma'} + h.c.)$ where spin up and down states correspond to the collinear and anti-collinear orientations of the second DW with respect to the first one.

Classically, if we consider a series of DWs pinned at a specific distance from each other, R^* , inside a wire then the total energy of the system is given by an XY-model

$$E = - \sum_i [J_1 \cos(\theta_{i+1} - \theta_i) + J_2 \cos(\theta_{i+2} - \theta_i)], \quad (11)$$

where only the nearest neighbour and next nearest neighbour interactions are included. Then we have $J_1 = -E_{int}(0, R^*)$ and $J_2 = -E_{int}(0, 2R^*)$. In general we can have J_1 and J_2 as either negative or positive and either $|J_1| > |J_2|$ or $|J_2| > |J_1|$. There are several possible set-ups admitting a simple solution. If $|J_1| \gg |J_2|$ then we have either an antiferromagnet: $\theta_{i+1} - \theta_i = \pi$ (for

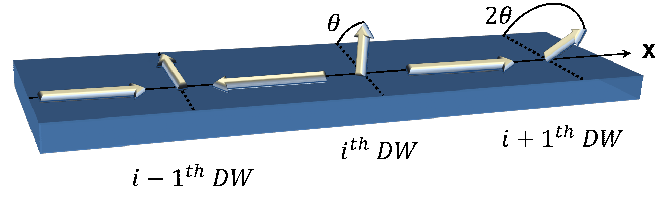


Figure 3 A schematic of the spiral structure of the DWs for $J_2 < 0$ and $|2J_2| > |J_1|$, the configuration of three of the DWs is shown. $\theta = \theta_{i+1} - \theta_i = \cos^{-1}[-J_1/2J_2]$. The DW orientation, defined at the centre of the DW, and the bulk magnetization are shown.

$J_1 < 0$), or a ferromagnetic arrangement $\theta_{i+1} - \theta_i = 0$ (for $J_1 > 0$). If we take $J_1 \rightarrow 0$ then we get two sublattices with either AFM or FM arrangements depending on the sign of J_2 . If we have $2J_2 < J_1$ and $2J_2 < -J_1$, i.e. $J_2 < 0$ and $|2J_2| > |J_1|$, then the model is minimized by $\cos[\theta_{i+1} - \theta_i] = -J_1/2J_2$. In this case we obtain a spiral structure of the DW orientations through the wire, see figure 3. Experimentally, similar spiral structures have been already observed [15, 16].

3 Conclusion In summary we have found that there exists an RKKY-like electron mediated interaction between DWs in a ferromagnetic sample that is long-range and oscillating. This interaction remains, *independent* of any current flowing through the system, in addition to previously found results for current mediated interactions in nanowires.

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